# CS 4375 Homework 5

April 12, 2022

Deadline for the first submission: **Apr-13-2022**.
All assignments **MUST** have your name, student ID, course name/number at the beginning of your documents.
Your homework **MUST** be submitted via eLearning with file format and name convention as follows:

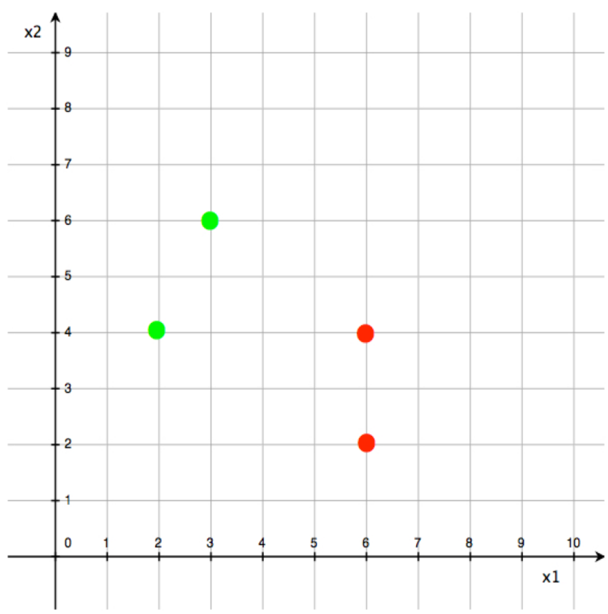
HW#_Name_writeup.**pdf** (for writing part)
HW#_Name_code.**ipynb** (for coding part)

If you have any questions, please contact me.

Q1 Given a dataset of two positive points (red points): $[6, 2]^T, [6, 4]^T$, and two negative points (green points): $[2, 4]^T, [3, 6]^T$, as shown in figure below, the problem of **a linearly separable SVM** can be formulated as the following form:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{p}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{G}\mathbf{x} \leq \mathbf{h} \end{aligned} \tag{1}$$

Write down the values of **Q**, **p**, **G**, and **h**.



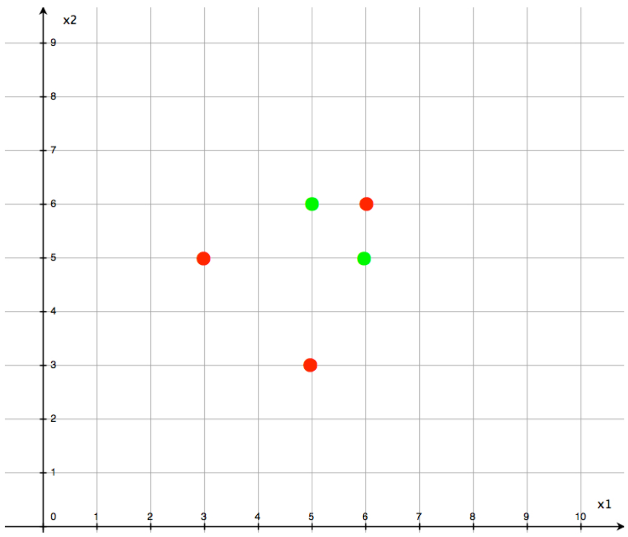

Q2 Modify the provided example code "example-code.py" to solve the **linearly separable SVM** problem in Q1 and output the following values:

1. The values of **w** and **b** that decide the separating line $\mathbf{w}.\mathbf{x} + \mathbf{b} = 0$.
2. The margin size.
3. The support vectors.
4. Predict the class labels of the points:
   $[3, 5]^T, [3, 4]^T, [4, 6]^T, [5, 4]^T, [5, 2]^T, [5, 6]^T$.

Q3 Given a dataset of three positive points (red points): $[3,5]^T, [5,3]^T, [6,6]^T$, and two negative points (green points): $[5,6]^T, [6,5]^T$, as shown in figure below, the problem of **a linearly non-separable SVM** can be formulated as the following form:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{p}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{G}\mathbf{x} \leq \mathbf{h} \end{aligned} \tag{2}$$

Write down the values of $\mathbf{Q}$, $\mathbf{p}$, $\mathbf{G}$, and $\mathbf{h}$.




Q4 Set the trade-off parameter $C = 1$. Modify the provided example code "example-code.py" to solve the **linearly non-separable SVM** problem in Q3 and output the following values:

1. The values of $\mathbf{w}$ and $\mathbf{b}$ that decide the separating line $\mathbf{w}.\mathbf{x} + \mathbf{b} = 0$.
2. The margin size.
3. The support vectors.
4. Predict the class labels of the points:
   $[3,4]^T, [3,6]^T, [4,3]^T, [4,5]^T, [5,4]^T, [5,2]^T, [5,6]^T, [6,4]^T, [6,7]^T, [7,5]^T, [7,6]^T$.

Q5 Set the trade-off parameter $C = 1000$. Use the same dataset in Q3 and the basis functions $\phi(\mathbf{x}) = [x_1, x_2, x_1^2, x_2^2, x_1 \cdot x_2]^T$, where $\mathbf{x} = [x_1, x_2]^T$. Further **modify the code implemented for Q4** to train **a nonlinear SVM model** and output the following values:

1. The values of $\mathbf{w}$ and $\mathbf{b}$ that decide the separating line $\mathbf{w}.\phi(\mathbf{x}) + \mathbf{b} = 0$.
2. The margin size.
3. The support vectors.
4. Predict the class labels of the points:
   $[3,4]^T, [3,6]^T, [4,3]^T, [4,5]^T, [5,4]^T, [5,2]^T, [5,6]^T, [6,4]^T, [6,7]^T, [7,5]^T, [7,6]^T$.