odes:-

$$\frac{\frac{\mu_{hnf}}{\mu_f}}{\frac{\rho_{hnf}}{\rho_f}} f''' + Nff'' - 2Nf'^2 + 2N + 2\frac{\frac{(\rho\beta)_{hnf}}{(\rho\beta)_f}}{\frac{\rho_{hnf}}{\rho_f}}\lambda\theta = 0$$

$$\frac{1}{Pr}\cdot\frac{1}{\frac{(\rho c_p)_{hnf}}{(\rho c_p)_f}}\left(\frac{k_{hnf}}{k_f} + \frac{4}{3}R\right)\theta'' + Nf\theta' - 4N\theta f' + \frac{\varepsilon}{Pr}\frac{\frac{k_{hnf}}{k_f}}{\frac{(\rho c_p)_{hnf}}{(\rho c_p)_f}}(A^* f'$$

$$+ B^*\theta) = 0$$

$f(0) = 0 \quad f'(0) = \varepsilon \quad \theta(0) = 1$

$f(\infty) = 1 \quad \theta(\infty) = 0.$

Initial guesses:-

$f = (1-\varepsilon)(e^{-x} - 1) + x$ — (1)

$\theta = e^{-x}.$ — (2)

$f' = (1-\varepsilon)(-e^{-x}) + 1.$

$f' = (-1+\varepsilon)e^{-x} + 1$ — (3)

$f(0) = 0$

Put in eq (1)

$f(0) = (1-\varepsilon)(e^{-0} - 1) + 0$

$= (1-\varepsilon)(1-1)$

$f(0) = 0$

$\boxed{0 = 0}$

$f'(0) = \varepsilon$ ②

Put $x=0$ in eq ③

$f'(0) = (-1+\varepsilon)e^{0} + 1$

$= (-1+\varepsilon)1 + 1$

$= \cancel{-1} + \varepsilon + \cancel{1}$

$\boxed{\varepsilon = \varepsilon}$

$f'(\infty) = 1$

Put $x=\infty$ in eq ③

$f'(\infty) = (-1+\varepsilon)e^{-\infty} + 1$ $\quad \because e^{\infty} = \infty$

$\because \frac{1}{\infty} = 0$

$f'(\infty) = (-1+\varepsilon)(0) + 1$

$f'(\infty = 1$

$\boxed{1 = 1}$

$\theta(0) = 1$

Put $x=0$ in eq ②

$\theta(0) = e^{0}$

$\boxed{1 = 1}$

$\theta(\infty) = 0$

Put $x=\infty$ in eq ②

$\theta(\infty) = e^{-\infty}$

$= \frac{1}{\infty}$

$\boxed{\theta(\infty) = 0}$

The Boundary condition are satify. Hence we choose the above given functions as initial guesses.

Auxiliary linear operator:-

$\mathcal{L}_f(f) = f''' - f' = 0 \quad - (4)$

$\mathcal{L}_\theta(\theta) = \theta'' - \theta = 0 \quad - (5)$

Now eq (4)

$f''' - f' = 0$

$(D^3 - D)f = 0$

$D^3 - D = 0$

$D(D^2 - 1) = 0$

$\boxed{D = 0} \quad D^2 - 1 = 0$

$(D+1)(D-1) = 0$

$D + 1 = 0 \quad D - 1 = 0$

$\boxed{D = -1} \quad \boxed{D = 1}$

$\mathcal{L}_f[C_1 + C_2 e^x + C_3 e^{-x}] = 0$

$\theta'' - \theta = 0$

$(D^2 - 1)\theta = 0$

$D^2 - 1 = 0$

$(D+1)(D-1) = 0$

$D + 1 = 0 \quad D - 1 = 0$

$\boxed{D = -1} \quad \boxed{D = 1}$

$\mathcal{L}_\theta[C_4 e^x + C_5 e^{-x}] = 0.$