MTAT.03.286: Design and Analysis of Algorithms **University of Tartu**

Homework assignment 1

Due date: October 4, 2021

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1. **40 points.** For each of the following, indicate whether $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, or $f(n) = \Theta(g(n))$. Justify your answer.

   (a) $f(n) = n^6$ and $g(n) = 5n^6 + 10n^4 - 120n^2 + 1000$;

   (d) $f(n) = n^{\frac{1}{5}}$ and $g(n) = \log_2 n \cdot \log_2 \log_2 n$.

   (b) $f(n) = 2^n$ and $g(n) = (\sqrt{n})^{\sqrt{n}}$;

   (c) $f(n) = n$ and $g(n) = (\sqrt{n})^{\log_2 n}$.

2. **30 points.** Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be an undirected, connected, finite graph with the weight function $w : \mathcal{E} \to \mathbb{R}^+$. Propose an algorithm that finds a circuit-free subgraph $\mathcal{H}(\mathcal{V}, \mathcal{E}_1)$ of $\mathcal{G}$ such that $\sum_{e \in \mathcal{E}_1} w(e)$ is maximal possible. Prove correctness of your algorithm and analyze its complexity.

3. **30 points.** Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be an undirected connected finite graph with the weight function $w : \mathcal{E} \to \mathbb{R}^+$. It is known that the weights of the edges in $\mathcal{E}$ are all different. Can $\mathcal{G}$ have *multiple different* minimum spanning trees? Justify your answer (provide an example or prove a converse).

4. **Bonus: 20 points.** Let $\mathcal{T}_1(\mathcal{V}, \mathcal{E}_1)$ and $\mathcal{T}_2(\mathcal{V}, \mathcal{E}_2)$ be two spanning trees of the undirected, connected, finite graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. Prove that for every edge $e \in \mathcal{E}_1 \backslash \mathcal{E}_2$ there exists an edge $e' \in \mathcal{E}_2 \backslash \mathcal{E}_1$, such that *each* of the edge sets

$$(\mathcal{E}_1 \cup \{e'\}) \backslash \{e\} \quad \text{and} \quad (\mathcal{E}_2 \cup \{e\}) \backslash \{e'\}$$

   defines a spanning tree.